

Small cosmological constant in seesaw mechanism with breaking down SUSY

V.V.Kiselev^{1,2} and S.A.Timofeev²

¹*Russian State Research Center “Institute for High Energy Physics”,
Pobeda 1, Protvino, Moscow Region, 142281, Russia
Fax: +7-4967-744937*

²*Moscow Institute of Physics and Technology, Institutskii per. 9, Dolgoprudnyi, Moscow Region, 141700, Russia*

The observed small value of cosmological constant can be naturally related with the scale of breaking down supersymmetry in agreement with other evaluations in particle physics.

PACS numbers: 98.80.-k

I. INTRODUCTION

Recent precise observations in cosmology [1, 2, 3, 4, 5] prefer for the model of flat Universe, which has the energy density composed by following three dominant components: baryons, dark matter and dark energy with fractions of energy approximately given by $\Omega_b \approx 0.04$, $\Omega_{dm} \approx 0.21$ and $\Omega_{de} \approx 0.75$, respectively. The dark energy is dynamically fitted by a quintessence [6], that is a slowly evolving scalar-field, whose potential energy imitates¹ the cosmological constant. The introduction of quintessence seems to be reasonable, since the cosmological constant itself [8] should give the energy density

$$\rho_\Lambda = \mu_\Lambda^4, \quad \text{at} \quad \mu_\Lambda \approx 0.25 \cdot 10^{-11} \text{ GeV}, \quad (1)$$

which leads to the artificially small scale in particle physics. The quintessence serves to produce such the scale due to the evolution of potential energy from natural values to the present-day point.

There is an alternative way to show that the small value of μ_Λ is not artificial but natural. Indeed, fluctuations between two vacuum-states with exact and broken down supersymmetry can result in small mixing and appearance of stationary vacuum level with the small cosmological constant. Thus, the cosmological constant could indicate the scale of supersymmetry breaking.

In Section II of present paper we assign the cosmological constant to the energy density of vacuum (zero-point) modes. If supersymmetry (SUSY) is exact the vacuum is flat, while breaking down SUSY results in a negative density of energy determined by the scale of SUSY breaking μ_X , and the vacuum state is given by Anti-de Sitter spacetime (AdS). We argue for the two vacua correlate. The decay of flat vacuum to AdS one [9] is forbidden due to the gravity effects [10], introducing a critical density of AdS state unreachable in supergravity [11]. Therefore, two vacuum-levels can get mixing, not the decay.

In Section III we consider static spherically-symmetric action of gravity and scalar field interpolating between two its positions in minima of potential with zero and negative values of energy density. Such the configuration

describes the bubble of AdS vacuum separated from the flat vacuum by the domain wall. We show that the domain wall does not propagate to infinity. Contrary, it has a finite size. We compare the situation with the case of gravity switched off as well as with the calculation of static energy describing the decay of flat vacuum if not forbidden. We estimate the size of bubble fluctuations, responsible for the mixing.

The mixing of two stationary vacuum-levels is studied in Section IV in cases of both thin and thick domain walls. The suppression of mixing matrix element leads to seesaw mechanism with small mixing angle [12], so that the observed small value of cosmological constant is naturally derived in terms of SUSY breaking scale μ_X and Planck mass.

The estimates in Section V show that thin domain walls correspond to low scale of SUSY breaking about $\mu_X \sim 10^4$ GeV, while thick domain walls give high scales of the order of $\mu_X \sim 10^{12-13}$ GeV.

In Section VI we formulate a model of superpotential, which allows us to demonstrate that thin domain walls correspond to gauge-mediated SUSY breaking as well as thick domain walls do to gravity-mediated SUSY breaking. Then, we evaluate the mixing angle in Section VII.

A connection of vacuum superposition to the problem of generations in the Standard Model (SM) of particle interactions is discussed in Section VIII, wherein we qualitatively map the way for the origin of three generations.

In Conclusion we summarize our results and focus on some further questions.

II. VACUUM MODES AND COSMOLOGICAL CONSTANT

The quantization of free bosonic and fermionic fields give hamiltonians in terms of creation and annihilation operators

$$\begin{aligned} E_B &= \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{1}{2} \{ \mathbf{a}_B^\dagger(\mathbf{k}) \mathbf{a}_B(\mathbf{k}) + \mathbf{a}_B(\mathbf{k}) \mathbf{a}_B^\dagger(\mathbf{k}) \} \omega_B(\mathbf{k}), \\ E_F &= \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{1}{2} \{ \mathbf{a}_F^\dagger(\mathbf{k}) \mathbf{a}_F(\mathbf{k}) - \mathbf{a}_F(\mathbf{k}) \mathbf{a}_F^\dagger(\mathbf{k}) \} \omega_F(\mathbf{k}), \end{aligned} \quad (2)$$

respectively, for each mode with $\omega(\mathbf{k}) = \sqrt{m^2 + \mathbf{k}^2}$.

¹ See review of quintessence phenomenology in [7].

The commutation and anti-commutation relations for bosons and fermions

$$[\mathbf{a}_B(\mathbf{k}), \mathbf{a}_B^\dagger(\mathbf{k}')] = \{\mathbf{a}_F(\mathbf{k}), \mathbf{a}_F^\dagger(\mathbf{k}')\} = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}'), \quad (3)$$

involve the delta-function at zero if $\mathbf{k} = \mathbf{k}'$. It is related with the spatial volume

$$(2\pi)^3 \delta(\mathbf{k}) \Big|_{\mathbf{k}=0} = \int d^3\mathbf{r} \cdot e^{i\mathbf{r} \cdot \mathbf{k}} \Big|_{\mathbf{k}=0} = \text{Volume}.$$

Then, the energy of single field-mode is given by the expression

$$E = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathbf{a}^\dagger(\mathbf{k}) \mathbf{a}(\mathbf{k}) \cdot \omega(\mathbf{k}) + (-1)^F \hat{\rho} \cdot \text{Volume}, \quad (4)$$

where $F = \{0, 1\}$ denotes the fermion number for bosonic or fermionic mode, correspondingly, while the energy density of zero-point mode $\hat{\rho}$ equals

$$\hat{\rho} = \frac{1}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \omega(\mathbf{k}). \quad (5)$$

The vacuum energy has the density²

$$\rho = \sum_{\text{modes}} (-1)^F \hat{\rho}. \quad (6)$$

At $\omega > 0$, the exact supersymmetry guarantees the followings:

- i) The number of bosonic modes is equal to the number of fermionic ones

$$I_W = \sum_{\text{modes}} (-1)^F = 0.$$

- ii) Masses of superpartners are equal to each other

$$m_B = m_F, \quad \Rightarrow \quad \omega_B(\mathbf{k}) = \omega_F(\mathbf{k}).$$

Therefore, the supersymmetric vacuum state $|\Phi_S\rangle$ has zero energy density $\rho_S = 0$ due to the contribution by the vacuum zero-point modes. The Witten's index I_W [13] counting for all physical modes would differ from zero in the supersymmetric theory [14], if one introduces different numbers of bosonic and fermionic modes with zero energy $\omega = 0$, but such the situation would correspond to the case, when, due to the conservation law for the number of unpaired zero-energy modes, the supersymmetry cannot be spontaneously broken in evident contradiction with observations.

A loss of balance between the modes produces a non-zero cosmological constant. The balance could be lost because of essential deviations from dispersion laws of free particles, that can appear due to a strong field dynamics beyond the asymptotically free region. Then, SUSY is broken down.

In ordinary schemes the SUSY breaking down is described by generation of different masses for superpartners at scales below μ_X , the characteristic energy of SUSY breaking. For instance, in the gauge-mediated scenario of SUSY breaking the superpartners of fields in the SM acquire masses of the order³

$$m \sim \frac{\alpha_g}{4\pi} \mu_X \ll \mu_X,$$

while the number of modes in the matter sector of theory is preserved, and the masses satisfy a rule of splitting

$$\sum_{\text{matter modes}} (-1)^F = 0, \quad \sum_{\text{matter modes}} (-1)^F m^2 = 0. \quad (7)$$

Effectively at scales below μ_X we put the dispersion law $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$. SUSY is restored at scales higher than μ_X . Then, the integration in the energy density of single vacuum-mode is actually cut off by μ_X because of exact cancelling by the superpartner contribution⁴, and we easily get

$$\begin{aligned} \hat{\rho} &= \frac{1}{2} \int_0^{\mu_X} \frac{k^2 dk}{(2\pi)^3} \sqrt{k^2 + m^2} \int d\Omega \\ &= \frac{2}{(16\pi)^2} m^4 (\sinh 4y - 4y), \end{aligned} \quad (8)$$

where

$$y = \text{arcsinh} \frac{\mu_X}{m} = \ln \left(\frac{\mu_X}{m} + \sqrt{\frac{\mu_X^2}{m^2} + 1} \right).$$

At $\frac{\mu_X}{m} \gg 1$ the leading contribution to the vacuum energy in the observable matter sector is about

$$\sum_{\text{matter modes}} (-1)^F \hat{\rho} \sim - \sum_{\text{matter modes}} (-1)^F m^4 \ln \frac{\mu_X}{m}, \quad (9)$$

since terms of the form μ_X^4 are cancelled due to the balance between the superpartner modes, i.e. Witten's index is equal to zero, while terms of the form $m^2 \mu_X^2$ nullify due to the sum rule for the mass splitting (7). The supercharge relation with the hamiltonian ensures the positivity of matter contribution to the vacuum energy (9), i.e. up to fine effects in higher orders of small ratio m/μ_X one should expect the following sum rule

$$\sum_{\text{matter modes}} (-1)^F m^4 \ln \frac{\mu_X}{m} < 0.$$

² Other procedures of quantization differ from the accepted way by an introduction of arbitrary renormalization of vacuum energy, that should involve some physical reasons. We do not see such the reasons for the subtractions.

³ See details in Weinberg's textbook [14].

⁴ See notes on the scheme of regularization in [16].

However, the direct breaking down SUSY at tree level in the minimal extension of SM contradicts with observations, since the mass sum rules (7) introduce too light superpartners for the particles of observable sector [14]. So, SUSY is broken in a hidden sector, which can carry *zero or nonzero quantum numbers of SM*, and the particles of observable sector acquire masses due to loops with particles from the hidden sector, that plays the role of messenger. The first scenario with messengers carrying nonzero SM charges refers to the gauge-mediated SUSY breaking, while the second possibility of sterile messengers does to gravity-mediated one. The masses of messengers are of the order of SUSY breaking scale, $m \sim \mu_X$. Hence, the contribution of hidden sector to the density of vacuum energy is dominant, $\rho \sim \pm \mu_X^4$. The sign can be certainly fixed, if one takes into account the result by W. Nahm, who algebraically found [15], that SUSY realization is forbidden in four-dimensional (4D) space-time with a positive density of vacuum energy, while it is permitted in 4D spacetime with a negative density of vacuum energy.

In the gravity sector, the SUSY breaking leads to two massless modes of graviton with spirality ± 2 as well as to two massive modes of graviton superpartner, the gravitino with spirality $\pm \frac{3}{2}$, while in addition the goldstino with spirality $\pm \frac{1}{2}$ becomes massive and it complements higher spirality modes of gravitino to the full set $\{\pm \frac{3}{2}, \pm \frac{1}{2}\}$. Therefore, the goldstino breaks the balance between the number of bosonic and fermionic modes in the gravity sector. Hence, the vacuum energy could gain the large negative contribution of two goldstino-modes

$$\sum_{\text{gravity}} (-1)^F \hat{\rho} \sim - \sum_{\text{goldstino}} \hat{\rho} \sim - \frac{1}{8\pi^2} \mu_X^4. \quad (10)$$

However, the goldstino is a composition of hidden sector spinor fields, i.e. its two modes are superpartners for the bosonic modes from the non-gravity sector. Therefore, the true value of vacuum energy is determined by the whole hidden sector as it has been matched above.

Thus, the vacuum modes in supergravity with SUSY broken below μ_X give the *negative cosmological term*, that corresponds to Anti-de Sitter spacetime. We denote such the state by $|\Phi_X\rangle$, which has got the negative energy density⁵ $\rho = -\rho_X \sim -\mu_X^4$.

Such the nature of vacuum energy assumes that two states $|\Phi_S\rangle$ and $|\Phi_X\rangle$ correlate, i.e. they are not completely independent, since the vacuum modes with momenta greater μ_X are common for both states. In other words, we can introduce the correlation length determined by the scale of SUSY breaking $\lambda_X = 1/\mu_X$, so that

dynamical processes at characteristic distances less than λ_X involve the correlation of two vacuum-states with zero and negative cosmological constants. The transitions between two states can have a status of whether we get the decay of unstable state into the stable one or mixing that leads to two stationary levels. The overlapping of two vacua is associated with the domain wall separating the bubble of lower-energy AdS-vacuum from the exterior of higher-energy flat vacuum. The process of decay is described in terms of bounce, the solution of 4D Euclidean spherically symmetric field-equations for a scalar field interpolating between local minima of its potential in the region of domain wall. The bounce determines the quasiclassical exponent of penetration between two levels of vacuum. Coleman and De Luccia [10] shown that the bounce is essentially modified by gravity that introduces a critical surface tension of domain wall, while S. Weinberg [11] found that the real surface density of energy exceeds the critical one in supergravity. Thus, the decay does not take place⁶. Therefore, we focus on stationary 3D spherically symmetric fluctuations of scalar field, that provide the mixing of two vacuum-states, if such the domain wall cannot evolve to spatial infinity.

III. STATIC ENERGY AND DOMAIN WALL

For fields independent of time, the action is converted to the static potential U^{stat} multiplied by the factor of total time

$$S = \int \mathcal{L} \sqrt{-g} \, d^4x \mapsto S^{\text{stat}} = -U^{\text{stat}} \int dt, \quad (11)$$

since the metric could be also written in the static form, too. In the case of spherical symmetry we get the metric

$$ds^2 = \tilde{B}(r) dt^2 - \frac{1}{B(r)} dr^2 - r^2(d\vartheta^2 - \sin^2 \vartheta d\varphi^2), \quad (12)$$

so that $\sqrt{-g} = r^2 \sin \vartheta \sqrt{\tilde{B}/B}$, while in the lagrangian of real scalar field $\phi(r)$ dependent of the radius

$$\mathcal{L}_f = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi),$$

the gradient term survives in the form

$$g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \mapsto -(\phi')^2 B, \quad (13)$$

where the prime denotes the derivative with respect to the distance r . Then the field equation reads as follows

$$\phi'' + u' \phi' + \frac{2}{r} \phi' = \frac{1}{B} \frac{\partial V}{\partial \phi} \quad (14)$$

⁵ At scales greater than μ_X , the dynamics is supersymmetric and, hence, its contribution to the cosmological constant is equal to zero, while at scales much less than μ_X contributions of other non-supersymmetric effects, like the gluon condensate in Quantum Chromodynamics etc., are negligibly small.

⁶ See some further arguments in [17].

with $u = \frac{1}{2} \ln(\tilde{B}B)$. The field equation allows the treatment in terms of Newtonian mechanics by the assignment of ϕ'' to the “acceleration” of “coordinate” ϕ , so that the force contains the “potential term” $\partial V/\partial \phi$ with “external parameter” B and the “friction” proportional to the “velocity” ϕ' . The friction coefficient $2/r$ enters because of the spatial dimension equal to 3, while the gravitation results in the friction if u' is positive, otherwise the gravitation causes the enlarging the acceleration.

The energy-momentum tensor $T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L}_f$ is composed by diagonal elements

$$\begin{aligned} T_t^t &= +\frac{1}{2} (\phi')^2 B + V, \\ T_r^r &= -\frac{1}{2} (\phi')^2 B + V, \end{aligned} \quad (15)$$

and $T_\theta^\theta = T_\varphi^\varphi = T_t^t$, which enter the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}.$$

Hence, due to the relation of scalar curvature with the trace of energy-momentum tensor

$$R = -8\pi G T,$$

the lagrangian of general relativity equal to

$$\mathcal{L}_{GR} = -\frac{R}{16\pi G},$$

and the static field lagrangian equal to

$$\mathcal{L}_f = -T_t^t,$$

we get the stationary energy depending on the size of sphere r_A inside of which the matter has a non-zero energy,

$$U^{\text{stat}}(r_A) = -4\pi \int_0^{r_A} V(\phi) \sqrt{\frac{\tilde{B}}{B}} r^2 dr. \quad (16)$$

The static potential equals zero if the scalar field is global, and it positioned at a local minimum of its potential with $V = 0$. If the local minimum at constant field is positioned at negative $V = -\rho_X$, then we arrive to Anti-de Sitter spacetime with

$$\tilde{B}_{\text{AdS}} = B_{\text{AdS}} = 1 + \frac{r^2}{\ell^2}, \quad \frac{1}{\ell^2} = \frac{8\pi G}{3} \rho_X, \quad (17)$$

and the *positive* static potential⁷

$$U_{\text{AdS}}^{\text{stat}} = \frac{4\pi}{3} r_A^3 \rho_X. \quad (18)$$

⁷ From (17) we conclude that the gravitational potential in AdS spacetime is given by $\varphi_{\text{AdS}} = r^2/2\ell^2 = 4\pi G \rho_X r^2/3$, and it is attractive in contrast to naive expectation for a dust cloud with negative energy. The reason is the large negative pressure in AdS vacuum $p = -\rho$, so the pressure makes a work, i.e. it produces the positive energy, which gravitates, too.

Let $\phi(r)$ be the solution, which interpolates between two local minima of potential with zero energy and negative $V = -\rho_X$. To the moment, we restrict ourselves by the consideration of thin domain wall, so that the field is essentially changing in a narrow layer of width δr near the sphere of radius r_A and $\delta r \ll r_A$. Then, the stationary potential is composed of two summands with integration in limits $[0, r_A]$ and $[r_A, r_A + \delta r]$ respectively,

$$U^{\text{stat}}(r_A) = \frac{4\pi}{3} r_A^3 \rho_X - 4\pi r_A^2 W_A, \quad (19)$$

where W_A determines the surface energy per unit area

$$W_A(r_A) = \frac{1}{r_A^2} \int_{r_A}^{r_A + \delta r} V(\phi) \sqrt{\frac{\tilde{B}}{B}} r^2 dr, \quad (20)$$

and it is positive if the local minima are separated by sufficiently high potential barrier.

At $\delta r \ll r_A \ll \ell$ we can safely neglect the contribution of friction in the field equation (14), since by the order of magnitude $\phi'' \sim \delta\phi/(\delta r)^2$, while the spatial term is at the level of $\phi'/r \sim \delta\phi/(\delta r)^2 \cdot \delta r/r_A \ll \phi''$, and the metric elements \tilde{B}, B are infinitely close to unit, so that $u'\phi' \sim r_A^2/\ell^2 \cdot 1/\delta r \cdot \delta\phi/\delta r \ll \phi''$. Therefore, in this limit the field equation does not involve any scale parameter external with respect to the potential V , and it reproduces the “kink” solution with the small value of r_A and the width δr determined by a mass parameter in V , since the field equation yields $1/(\delta r)^2 \sim \delta V/(\delta\phi)^2 \sim \partial^2 V/\partial\phi^2$. Note, that the gradient contribution to the energy density T_t^t equals the potential term [10]. The kink sets the distribution of matter determining the behavior of metric. Thus, the thin domain wall can be established in the limit of small bubble.

At $\delta r \ll r_A \sim \ell$ the gravitational contribution to the field equation has two regimes. At the inner surface of domain wall, i.e. at the edge of AdS spacetime, the metric elements \tilde{B}, B are about unit and $u' > 0$ at $u' \sim r_A/\ell^2 \sim 1/r_A$, so that one could neglect its contribution as well as the friction term. Inside the wall the metric elements \tilde{B}, B can rapidly fall to unit and $u' < 0$ at $u' \sim 1/\delta r$, so that $u'\phi' \sim \phi''$ and the gravity term accelerates the evolution of field from the negative minimum to positive one, if the field evolves from a small value to larger one. Therefore, the surface tension W_A can depend on the bubble size, but the width of the domain wall still remains at the same order as it was at small r_A , that preserves the magnitude of W_A , too. In this region of bubble size the gradient term in the energy density is comparable to the potential.

We can evaluate the surface tension W_A by setting $\tilde{B} \sim B$ and $V \sim (\phi')^2$, so that $W_A \sim \int \sqrt{V} \phi' dr \sim \int \sqrt{V} d\phi$, while in the supersymmetric theory with the chiral superfield the potential is determined by the superpotential f as $V = |\partial f/\partial\phi|^2$, hence, $W_A \sim |f_0|$, where f_0 is the su-

perpotential value at the vacuum⁸. In supergravity the *negative* vacuum energy at the extremal of superpotential is assigned to the superpotential itself in the linear order in Newtonian constant G

$$\rho_X = 24\pi G |f_0|^2, \quad (21)$$

that yields

$$W_A \sim m_{\text{Pl}} \mu_X^2, \quad (22)$$

where $m_{\text{Pl}} = 1/\sqrt{G} \sim 10^{19}$ GeV is the Planck mass.

At $r_A \gg \ell$ the metric elements at the edge of AdS spacetime become large $\tilde{B} \sim B \sim r_A^2/\ell^2 \gg 1$, and the gravity term in the left hand side of field equation (14) can still be essential, since at $\delta u \sim 1$ we estimate $u'\phi' \sim \delta\phi/(\delta r)^2 \sim \phi''$, while condition $B \gg 1$ leads to suppression of gradient term in the energy as well as to more thick domain wall because of the approximation $B \cdot \phi'' \sim \partial V/\partial\phi$, hence, $(\phi')^2 \ll V$ and $1/(\delta r)^2 \sim \partial^2 V/\partial\phi^2 \cdot \ell^2/r_A^2$. Note, that the width of domain wall essentially exceeds its “natural” value δr_0 determined by the parameters of potential V at small r_A , and it *linearly* grows with r_A like $\delta r \sim \delta r_0 \cdot r_A/\ell$. Switching the regimes in W_A versus r_A depends on the parameters of potential. The simple example with $W_A = W_A^0 [1 + \frac{r_A}{b\ell} (1 + \tanh\{\frac{r_A}{b\ell} - b'\})]$ at $b' \gg 1$ allows us to draw a conclusion on the critical behavior of W_A versus the scale of switch $r_A \sim b\ell$, as it is depicted in Fig. 1, that shows the static potential U^{stat} . Moreover, at large $r_A \gg \ell$ the domain wall could disintegrate at all.

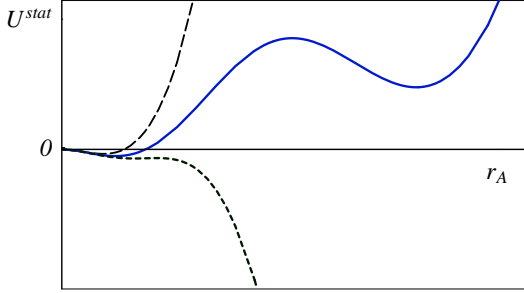


FIG. 1: The static potential of bubble with the domain wall versus the bubble radius at different behavior of surface tension: naively constant W_A^0 (long-dashed curve), with a large scale of switching the regime (solid curve) and a low scale of switching (dotted curve). The low scale of switching is not realistic, since it should mean the opportunity of domain-wall motion to infinity, i.e. the decay, that is forbidden (see text).

We assume that the critical scale is large enough in order to provide the materialization of bubble with zero

static potential. Then, the bubble can arise in the vacuum with zero density of energy. The characteristic size of such the bubble is given by solving $U^{\text{stat}} = 0$, that gives

$$r_A = \frac{3W_A}{\rho_X} \sim \ell. \quad (23)$$

The materialization of bubble in the flat vacuum results in the instability, since it takes place at the size of r_A , that is not positioned at the local minimum of static potential: the domain wall begins to move to the bubble center (see Fig. 1). Furthermore, the zero size of bubble is also unstable: the flat vacuum suffers from fluctuations due to the bubbles of AdS vacuum.

This situation is opposite to the case of switching off the gravity. Indeed, the elimination of gravitational action results in the static potential of bubble

$$U_0^{\text{stat}} = -\frac{4\pi}{3} r_A^3 \rho_X + 4\pi r_A^2 \tilde{W}_A,$$

where

$$\tilde{W}_A(r_A) = \frac{1}{r_A^2} \int_{r_A}^{r_A + \delta r} \left\{ V(\phi) + \frac{1}{2} (\phi')^2 \right\} r^2 dr.$$

This static potential formally has the opposite sign in comparison with (19). Therefore, the domain wall can materialize after the penetration through the potential barrier, but it will move to spatial infinity, that means the *decay* of flat vacuum to the AdS one. The description of penetration in the presence of gravity was considered by Coleman and De Luccia [10], involving the Euclidean action and spherical symmetry. So, the critical surface tension was found, and in fact [11] the decay is forbidden, since the tension exceeds the critical value⁹.

Indeed, at weak gravitational field, i.e. at $G \rightarrow 0$, one can easily evaluate the static energy E^{stat} by summing up

- the energy of AdS-vacuum bubble

$$M_b = -\frac{4}{3} \pi r_A^3 \rho_X,$$

- the energy of domain wall $M_{\text{dw}} = 4\pi r_A^2 \tilde{W}_A$,
- the gravitational potential of wall-bubble interaction

$$\varphi_{\text{AdS}} M_{\text{dw}} = \varphi_{\text{AdS}} 4\pi r_A^2 \tilde{W}_A = \frac{16}{3} \pi^2 G r_A^2 \rho_X \tilde{W}_A,$$

⁸ The derivation closely follows the original study by S. Weinberg in [11].

⁹ This fact supports our previous assumption on the large scale of switching the regimes in the surface tension W_A .

- the gravitation of thin domain wall itself

$$\int \varphi_{\text{dw}} dM_{\text{dw}} = -\frac{G}{r_A} \int M_{\text{dw}} dM_{\text{dw}} = -8\pi^2 G r_A^4 \widetilde{W}_A^2,$$

that yields

$$E^{\text{stat}} \approx -\frac{4\pi}{3} r_A^3 \rho_X + 4\pi r_A^2 \widetilde{W}_A \left(1 + \frac{1}{2} \frac{r_A^2}{\ell^2}\right) - 8\pi^2 G r_A^3 \widetilde{W}_A^2.$$

Beyond the weak-field approximation in [11] S.Weinberg found

$$E^{\text{stat}} = -\frac{4\pi}{3} r_A^3 \rho_X + 4\pi r_A^2 \widetilde{W}_A \sqrt{1 + \frac{r_A^2}{\ell^2}} - 8\pi^2 G r_A^3 \widetilde{W}_A^2,$$

where the only modification of wall-bubble term is related with the strict definition of thin domain-wall density of energy in terms of Dirac delta-function

$$\rho_{\text{dw}} = \frac{\widetilde{W}_A}{\sqrt{B}} \delta(r - r_A)$$

with $B = B_{\text{AdS}}$, that preserves the invariance under reparameterizations of radius. Such the static energy is the mass determining the Schwarzschild metric beyond the bubble and domain wall, so it has nothing with the static value of action, $U^{\text{stat}} \neq E^{\text{stat}}$. It is the easy task to find that E^{stat} nullifies at

$$r_A = \frac{r_A^0}{1 - \left(\frac{r_A^0}{2\ell}\right)^2}, \quad \text{at} \quad r_A^0 = \frac{3\widetilde{W}_A}{\rho_X}.$$

Therefore, $r_A^0 < 2\ell$ and the critical density is given by $\rho_X^c = 6\pi G \widetilde{W}_A^2$. S.Weinberg shown that the surface tension is constrained by the superpotential as $\widetilde{W}_A \geq 2|f_0|$. Thus, $\rho_X^c > 24\pi G |f_0|^2 = \rho_X$, and the flat vacuum cannot decay to the AdS one. We have to stress two points. First, the above conclusions on the behavior of E^{stat} is made at *exactly constant surface tension* \widetilde{W}_A . Second, at arbitrary \widetilde{W}_A , nullifying the static energy E^{stat} describes the materialization of bubble, which is strictly considered in [10] in terms of Euclidean 4D-symmetric action, so that one gets the standard quasiclassical calculation of **bounce**. Contrary, the static action corresponds to unstable fluctuations usually called **sphalerons**¹⁰, which are considered in 3D space. Such the bounce and sphaleron are generally *different* classical solutions, so certainly $E^{\text{stat}} \neq U^{\text{stat}}$.

As we have just shown the gravity induces the materialization of bubble not propagating to infinity, that means the *mixing* of two levels, but *not the decay*.

Thus, due to the unstable bubbles the vacua are not eigenstates of true hamiltonian.

IV. TWO LEVEL SYSTEM

Consider the quantum system of two stationary vacuum-levels within the domain wall, which is described by the hamiltonian density $\mathcal{H} = H_{\text{vac.}} / \text{Volume}$,

$$\begin{aligned} \mathcal{H} = & -\rho_X |\Phi_X\rangle \langle \Phi_X| + \rho_S |\Phi_S\rangle \langle \Phi_S| \\ & + \tilde{\rho} \{ |\Phi_X\rangle \langle \Phi_S| + |\Phi_S\rangle \langle \Phi_X| \}, \end{aligned} \quad (24)$$

where $\rho_X \sim \mu_X^4$ in the AdS vacuum with broken SUSY, while in the supersymmetric vacuum $\rho_S = 0$. We define global complex phases of states, so that the quantity $\tilde{\rho}$ takes a real positive value. The transition is associated with fluctuations described by the domain wall corresponding to the overlapping region of states. The bubble of AdS vacuum has the size $r_A \sim \ell$, the domain wall has a width δr . Let us, first, evaluate the width of domain wall δr in various cases and, second, estimate the mixing matrix element $\tilde{\rho} = \langle \Phi_S | \mathcal{H} | \Phi_X \rangle$.

A. Thin domain wall

If the domain wall is thin, its mass is given by the expression $M_{\text{dw}} = 4\pi r_A^2 W_A \sim 4\pi \ell^2 \delta r V_0$, where V_0 is the characteristic height of potential barrier inside the wall. This mass is compensated by the negative mass of bubble $M_b = -4\pi r_A^3 \rho_X / 3 \sim -\mu_X^4 \ell^3$, so that under $\ell \sim m_{\text{P1}} / \mu_X^2$ we get

$$\delta r \cdot V_0 \sim \ell \rho_X \sim m_{\text{P1}} \mu_X^2. \quad (25)$$

Furthermore, for the chiral superfield, the potential is defined by $V = |\partial f / \partial \phi|^2$, where in the linear order in G the superpotential f_0 at stationary point is related with the negative density of vacuum energy by (21), that gives

$$f_0 \sim m_{\text{P1}} \mu_X^2 \Rightarrow V_0 \sim \frac{f_0^2}{(\delta \phi)^2} \sim \frac{m_{\text{P1}}^2 \mu_X^4}{(\delta \phi)^2}, \quad (26)$$

where $\delta \phi$ is the characteristic change of field in the domain wall, i.e. the “distance” between two extremal points of the field. Hence, we evaluate the width of domain wall in terms of evolution change of the field,

$$\delta r \sim \frac{(\delta \phi)^2}{m_{\text{P1}} \mu_X^2}. \quad (27)$$

Putting $\delta r \ll r_A \sim \ell$, we find

$$\delta \phi \ll m_{\text{P1}}. \quad (28)$$

Therefore, the domain wall is thin, if the field dynamics is essentially sub-Planckian.

For instance, we get

$$\delta \phi \sim \mu_X \Rightarrow \delta r \sim \frac{1}{m_{\text{P1}}}, \quad (29)$$

$$\delta \phi \sim \sqrt{m_{\text{P1}} \mu_X} \Rightarrow \delta r \sim \lambda_X = \frac{1}{\mu_X}. \quad (30)$$

¹⁰ More strictly, sphalerons actualize a minimal value of potential barrier.

The case of $\delta r \sim \lambda_X$ looks the most natural situation, since the domain wall has the size of correlation length of two vacua. At $\sqrt{m_{\text{P1}} \mu_X} \ll \delta\phi \ll m_{\text{P1}}$ the domain wall becomes thick with respect to the correlation length λ_X . This case requires especial consideration.

The correlation energy of two states can be estimated in terms of mixing density of energy multiplied by the volume of the bubble,

$$E_{\text{corr.}} \sim \tilde{\rho} \cdot \ell^3. \quad (31)$$

On the other hand, it is determined by the energy in the overlapping region restricted by the correlation length λ_X , i.e. in the element of thin domain wall with the area of the order of λ_X^2 . Hence, $E_{\text{corr.}}$ is given by the surface tension $W_A \sim \delta r \cdot V_0$ in the area of correlation

$$E_{\text{corr.}} \sim W_A \cdot \lambda_X^2. \quad (32)$$

Value (32) gives the energy of domain wall in the beginning of materialization at $r_A \mapsto \lambda_X$.

Therefore, under $W_A \sim f_0 \sim m_{\text{P1}} \mu_X^2$ we get the estimate

$$\tilde{\rho} \sim \frac{\mu_X^2}{\ell^2} \sim \frac{\mu_X^6}{m_{\text{P1}}^2}, \quad (33)$$

implying $\tilde{\rho} \ll \rho_X$.

At $\sqrt{m_{\text{P1}} \mu_X} \ll \delta\phi \ll m_{\text{P1}}$ the correlation energy is determined by the height of potential barrier within the correlation volume $E_{\text{corr.}} \sim V_0 \cdot \lambda_X^3$, that yields $\tilde{\rho} \ll \mu_X^6/m_{\text{P1}}^2$ satisfying the same condition $\tilde{\rho} \ll \rho_X$ as above.

B. Thick domain wall

The mass of thick domain wall is estimated in terms of characteristic height of the barrier $M_{\text{dw}} \sim (\delta r)^3 V_0$, that is opposite to the mass of bubble with size $r_A \sim \ell$, where the energy density is negative. So,

$$(\delta r)^3 \cdot V_0 \sim \ell^3 \rho_X, \quad (34)$$

that leads to

$$(\delta r)^3 \sim \frac{m_{\text{P1}} (\delta\phi)^2}{\mu_X^6}. \quad (35)$$

Putting $\delta r \gg \ell$, we get

$$\delta\phi \gg m_{\text{P1}}, \quad (36)$$

and the dynamics of thick domain wall is related with super-Planckian fields.

The correlation energy is determined by the dominant volume of thick domain wall

$$E_{\text{corr.}}^{\text{thick}} \sim \tilde{\rho} \cdot (\delta r)^3, \quad (37)$$

which is equal to the characteristic energy inside the wall within the correlation volume

$$E_{\text{corr.}}^{\text{thick}} \sim V_0 \cdot \lambda_X^3. \quad (38)$$

Therefore, we get

$$\tilde{\rho} \sim \frac{m_{\text{P1}} \mu_X^7}{(\delta\phi)^4}, \quad (39)$$

and again $\tilde{\rho} \ll \rho_X$ due to (36) and $\mu_X \ll m_{\text{P1}}$.

C. Seesaw mechanism

We have just draw the conclusion that the matrix of two-level hamiltonian of vacuum has the form

$$\mathcal{H} = \begin{pmatrix} -\rho_X & \tilde{\rho} \\ \tilde{\rho} & 0 \end{pmatrix} \quad \text{at} \quad \tilde{\rho} \ll \rho_X, \quad (40)$$

so that such the texture is well known in the particle phenomenology as the “seesaw mechanism” for describing the mixing of charged currents, for instance [12]. Some applications of seesaw mechanism to the cosmological constant problem have been recently considered in [18], while the small scale in the quintessence dynamics generated due to seesaw, has been studied in [19].

The eigenvalues of (40) are equal to

$$\rho_\Lambda = -\frac{1}{2} \left(\rho_X \pm \sqrt{\rho_X^2 + 4\tilde{\rho}^2} \right), \quad (41)$$

and due to $\tilde{\rho} \ll \rho_X$ they are reduced to

$$\begin{aligned} \rho_\Lambda^{\text{dS}} &\approx \frac{\tilde{\rho}^2}{\rho_X}, \\ \rho_\Lambda^{\text{AdS}} &\approx -\rho_X, \end{aligned} \quad (42)$$

that corresponds to expanding de Sitter (dS) universe and collapsing AdS universe. Both vacua are stationary levels with no mixing or decay. We are certainly living in the Universe with the dS vacuum.

The eigenstates are described by superposition of initial non-stationary vacua

$$\begin{aligned} |\text{vac}\rangle &= \cos \theta_K |\Phi_S\rangle + \sin \theta_K |\Phi_X\rangle, \\ |\text{vac}'\rangle &= \cos \theta_K |\Phi_X\rangle - \sin \theta_K |\Phi_S\rangle, \end{aligned} \quad (43)$$

with the mixing angle¹¹ equal to

$$\tan 2\theta_K = \frac{2\tilde{\rho}}{\rho_X}, \quad (44)$$

well approximated by

$$\sin \theta_K \approx \frac{\tilde{\rho}}{\rho_X} \ll 1. \quad (45)$$

Thus, we arrive to the analysis of cosmological constant in different schemes of fluctuations in the region of overlapping the two initial vacuum-states, i.e. the domain wall.

¹¹ The subscript “K” is the abbreviation of Russian “kachely” translated as “seesaw”.

V. ESTIMATES

The thin domain wall determines

$$\rho_{\Lambda}^{\text{dS}} \sim \frac{\mu_X^8}{m_{\text{P1}}^4}, \quad (46)$$

and due to $\rho_{\Lambda} = \mu_{\Lambda}^4$ we get the estimate¹²

$$\mu_X \sim \sqrt{m_{\text{P1}} \mu_{\Lambda}} \sim 10^4 \text{ GeV}. \quad (47)$$

Thus, the thin domain wall is relevant to the low scale of SUSY breaking.

For the thick domain walls we arrive to the estimate

$$\rho_{\Lambda}^{\text{dS}} \sim \frac{m_{\text{P1}}^2 \mu_X^{10}}{(\delta\phi)^8}. \quad (48)$$

Then, the comparison with observed cosmological constant gives rough estimates at various evolution change of field, for example,

$$\begin{aligned} \delta\phi \sim \frac{m_{\text{P1}}^2}{\mu_X} &\Rightarrow \mu_X \sim 10^{12} \text{ GeV}, \\ \delta\phi \sim \frac{m_{\text{P1}}^2}{\mu_X} \sqrt{\frac{m_{\text{P1}}}{\mu_X}} &\Rightarrow \mu_X \sim 10^{13} \text{ GeV}. \end{aligned} \quad (49)$$

Therefore, thick domain walls are relevant to the high scale of SUSY breaking.

The relation of SUSY breaking scenario with different regimes of domain wall fluctuations can be clarified by considering some typical properties of scalar field potential.

VI. MODEL POTENTIAL

For simplicity, consider the real scalar field and AdS vacuum density modelled by a single fermionic mode of formula (8)

$$\rho_X \mapsto \hat{\rho}. \quad (50)$$

Introduce the field \mathcal{M} defined as the bottom boundary of integration versus the vacuum modes in the energy density,

$$\hat{\rho}(\mathcal{M}) = \int_{\mathcal{M}}^{\mu_X} \frac{k^2 dk}{(2\pi)^2} \sqrt{k^2 + m^2}. \quad (51)$$

This field should be physical, since it describes the generation of SUSY breaking. At $\mathcal{M} = \mu_X$, SUSY is exact, while at $\mathcal{M} = 0$ we get $\rho_X = \hat{\rho}(0)$ and SUSY is broken down.

The field \mathcal{M} is constrained by limits $\mathcal{M} \in [0, \mu_X]$. In addition, the above definition can involve non-canonic kinetic energy. Therefore, \mathcal{M} is actually expressed in terms of canonic scalar field ϕ , i.e. $\mathcal{M} = \mathcal{M}(\phi)$.

Let us assign the superpotential¹³ of ϕ by supergravity relation

$$f^2(\phi) = \frac{1}{24\pi G} \hat{\rho}(\mathcal{M}) \sim m_{\text{P1}}^2 \mu_X^4. \quad (52)$$

Then, the potential is given by the expression¹⁴

$$V(\phi) = \left| \frac{\partial f}{\partial \phi} \right|^2, \quad (53)$$

which is calculated as the derivative of composite function. This fact causes three important points.

First, at $\mathcal{M} \rightarrow \mu_X$ the vacuum density of energy nullifies $\hat{\rho} \sim \mu_X^4 - \mathcal{M}^4$ at $m \ll \mu_X$ or $\hat{\rho} \sim \mu_X^3 - \mathcal{M}^3$ at $m \gg \mu_X$, while actually $m \sim \mu_X$, so that anyway the superpotential behaves like

$$f \sim \sqrt{1 - \frac{\mathcal{M}}{\mu_X}},$$

and there is the singularity

$$\frac{\partial f}{\partial \mathcal{M}} \sim \frac{1}{\sqrt{1 - \frac{\mathcal{M}}{\mu_X}}}.$$

The simplest way to avoid the singularity is to postulate an appropriate behavior of derivative for \mathcal{M} with respect to ϕ like

$$\frac{d\mathcal{M}}{d\phi} \sim 1 - \frac{\mathcal{M}}{\mu_X}. \quad (54)$$

Then, the potential will be regular at its local minimum corresponding to the flat vacuum with $\rho_S = 0$. Solution of (54) is given by the exponential potential. In more general form, we put

$$\left(\frac{\mathcal{M}}{\mu_X} \right)^{\nu} = 1 - \exp \left\{ -\frac{\phi^2}{\tilde{m}^2} [1 + \mathcal{C}(\phi)] \right\}, \quad (55)$$

where \tilde{m} is a scale, ν is integer, while $\mathcal{C}(\phi)$ is a polynomial function, introducing corrections to the quadratic dependence of the exponent argument versus the field. The quadratic behavior is introduced in order to preserve the limits of \mathcal{M} as well as the invariance under $\phi \leftrightarrow -\phi$, for the sake of simplicity.

¹² Estimate (47) was obtained by T.Banks in [20] in other way of physical argumentation for the mechanism of SUSY breaking.

¹³ It is important to emphasize that we deal with the low-energy effective potential of scalar field, that should be considered as the correction to a true superpotential safely neglected at such values of field, where the introduced correction is essential.

¹⁴ Remember, we deal with the real field.

Second, at $\mathcal{M} \rightarrow 0$ the vacuum density tends to its AdS value as $\hat{\rho} \sim \mu_X^4 - m' \cdot \mathcal{M}^3$, so that the superpotential acquires the dependence in the form¹⁵

$$f \sim 1 - \tilde{b} \frac{\mathcal{M}^3}{\mu_X^3}. \quad (56)$$

At this point, SUSY is broken, hence $\partial f / \partial \phi \neq 0$, that can be easily satisfied if

$$\mathcal{M}^3 \sim \phi \rightarrow 0. \quad (57)$$

This condition is provided by ansatz (55) at $\nu = 6$, since $\mathcal{C} \rightarrow 0$ at $\phi \rightarrow 0$.

Third, the vacuum energy in the scalar sector given by V at $\phi \rightarrow 0$ is modified by supergravity [14]

$$\rho \rightarrow \left| \frac{\partial f}{\partial \phi} \right|^2 - 24\pi G f^2(0) = \left| \frac{\partial f}{\partial \phi} \right|^2 - \rho_X. \quad (58)$$

To preserve the AdS spacetime we should require

$$\left| \frac{\partial f}{\partial \phi} \right|^2 \lesssim 24\pi G f^2, \quad \text{at } \phi \rightarrow 0,$$

or approximately

$$\frac{m_{\text{P}1}^2}{\tilde{m}^2} \mu_X^4 \lesssim \mu_X^4,$$

that can be satisfied by putting $\tilde{m} = \tilde{m}_{\text{thin}}$, where

$$\tilde{m}_{\text{thin}} = \frac{m_{\text{P}1}}{\gamma}, \quad (59)$$

so that $\gamma^2 \sim \mu_X/m \sim 1$ with m being the mass in the single vacuum density of energy (51), and such value of γ provides the correct expectation $V(0) \sim \mu_X^4$, that is appropriate for thin domain walls as we will see below, since it provides the sub-Planckian changes of field in the domain wall.

At $V(0) \ll \mu_X^4$, one could expect that $V(0)$ is suppressed by gravitational constant G , and hence,

$$\tilde{m}_{\text{thick}} \sim \frac{m_{\text{P}1}^{n+1}}{\mu_X^n} \gg m_{\text{P}1}, \quad \text{at integer } n > 0, \quad (60)$$

that is appropriate for thick domain walls with super-Planckian changes of field.

So, the potential model in (55) is almost defined. The only uncertainty is entered through integer n and function $\mathcal{C}(\phi)$, which properties are related with the dynamics of SUSY breaking down.

A. Gauge-mediated SUSY breaking

The correction function could look as the expansion in inverse $\phi_g \sim \mu_X$ determined by a strong-field interaction in the gauge sector, so that to the leading order one could expect

$$\mathcal{C}(\phi) \mapsto \frac{\phi^2}{\phi_g^2}. \quad (61)$$

The complete potential energy of the field, including linear G -corrections from supergravity, has the form¹⁶

$$U(\phi) = V(\phi) - 24\pi G \left(f(\phi) - \frac{\phi}{3} \frac{\partial f}{\partial \phi} \right)^2 + \frac{16\pi}{3} G \phi^2 \left(\frac{\partial f}{\partial \phi} \right)^2. \quad (62)$$

Characteristic behavior of quantity (62) under (61) is shown in Fig. 2.

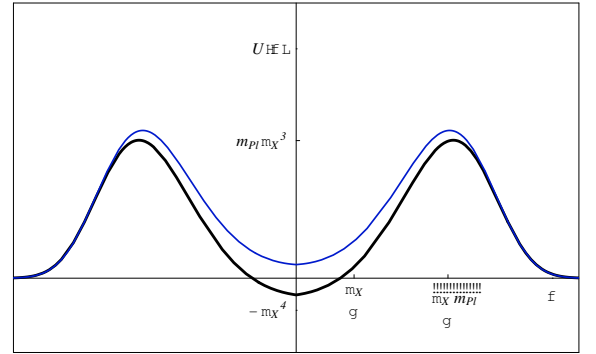


FIG. 2: The potential of scalar field $U(\phi)$ in the gauge-mediated scheme of SUSY breaking at $\phi_g = \mu_X/\gamma$ and $\tilde{m} \sim m_{\text{P}1}/\gamma$. The upper curve shows the potential with no supergravity corrections.

It is clear that $U(\phi)$ starts to rapidly grow from $U(0)$ at $\phi \sim \phi_g \sim \mu_X$, where $\mathcal{C}(\phi)$ effectively becomes to dominate with respect to unit. The potential begins to fall at $\phi \sim \sqrt{\tilde{m}} \phi_g \sim \sqrt{m_{\text{P}1}} \mu_X \gg \mu_X$. Then, the characteristic change of field between two minima of potential¹⁷ is about $\delta\phi \sim \sqrt{m_{\text{P}1}} \mu_X$, which corresponds to thin domain wall.

Thus, the thin domain wall is relevant to the gauge-mediated SUSY breaking at low scales $\mu_X \sim 10^4$ GeV.

¹⁵ The relation between the superpotential and density of vacuum energy in general involves higher orders in Newtonian constant, so that sub-leading terms can induce a linear correction to the cubic dependence as

$$\frac{\mathcal{M}^3}{\mu_X^3} + \tilde{b} \frac{\mathcal{M}}{\mu_X} \frac{\tilde{\mu}^2}{m_{\text{P}1}^2},$$

that slowly modify the potential behavior at $\phi \rightarrow 0$, which is not important for our purposes.

¹⁶ See, for instance, [14].

¹⁷ The method of potential reconstruction in the model does not allow us to make certain conclusions about an actual potential behavior at infinity $\phi \rightarrow \infty$ because it can be not related with the energy of vacuum modes. Therefore, the true form of potential far away from local minima are not shown in Fig. 2.

B. Gravity-mediated SUSY breaking

If the gravity is responsible for the transition of SUSY breaking to the observed matter sector, the expansion of \mathcal{C} is composed versus powers of Newtonian constant, i.e. in the inverse Planck mass. Therefore, to the leading order one expects

$$\mathcal{C}(\phi) \mapsto \frac{\bar{\gamma}^2 \phi^2}{m_{\text{Pl}}^2}, \quad \text{at } \bar{\gamma} \sim 1. \quad (63)$$

The leading term depends on the mass scale \tilde{m}_{thick} , which can be estimated by

$$\frac{\phi^2}{\tilde{m}_{\text{thick}}^2} \mapsto \frac{\phi^2 \phi_{\text{GR}}^2}{m_{\text{Pl}}^4}, \quad (64)$$

where $\phi_{\text{GR}} \ll \mu_X$ denotes the characteristic scale of observed fields or superpartner masses, which is composed by breaking scale μ_X , and it includes powers of inverse Planck mass, too. Therefore,

$$\tilde{m}_{\text{thick}} = \frac{m_{\text{Pl}}^2}{\phi_{\text{GR}}},$$

while

$$\delta\phi \sim \sqrt{\tilde{m}_{\text{thick}} m_{\text{Pl}}}.$$

For instance, at $\phi_{\text{GR}} \sim \mu_X^2/m_{\text{Pl}} \sim \sqrt{G} \mu_X^2$ we find the distance between fields fitted to the minima of potential

$$\delta\phi \sim \frac{m_{\text{Pl}}^2}{\mu_X},$$

while $\phi_{\text{GR}} \sim \mu_X^3/m_{\text{Pl}}^2 \sim G \mu_X^3$ corresponds to

$$\delta\phi \sim \frac{m_{\text{Pl}}^2}{\mu_X} \sqrt{\frac{m_{\text{Pl}}}{\mu_X}}.$$

Both above cases of ϕ_{GR} represent two known versions of standard scenario for the gravity-mediated SUSY breaking [14].

Since the field is exposed to super-Planckian changes, we deal with thick domain walls in the gravity-mediated SUSY breaking at high scales about 10^{12-13} GeV.

To the end of this Section, we especially emphasize that at super-Planckian changes of field in thick domain walls, the height of potential barrier takes the values much less than the energy density of AdS vacuum, $V_0 \ll \mu_X^4$. Therefore, one should control the dimensionless parameters like $\bar{\gamma}$ in order to get positive values of actual potential (62) within the wall. In this respect, one can see the role of presented potential as a toy model, that serves to demonstrate some general features of scale dependence in the problem. In practice, the form of true potential is strongly depends of the field contents in the theory. Moreover, remember that we have accented the attention on the nonperturbative low-energy contribution and neglected a tree potential.

VII. ANGLE θ_K

The mixing angle of two levels θ_K takes different values depending on the scenario of SUSY breaking.

For thin domain wall we get

$$\theta_K \approx \frac{\tilde{\rho}}{\rho_X} \sim \frac{\mu_X^2}{m_{\text{Pl}}^2} \sim \frac{\mu_\Lambda}{m_{\text{Pl}}}. \quad (65)$$

Therefore, its value is certainly fixed by present day data on the cosmological constant, $\theta_K \sim 10^{-30}$.

In contrast, for thick domain walls we write down

$$\theta_K \approx \sqrt{\frac{\tilde{\rho}^2}{\rho_X^2}} = \sqrt{\frac{\rho_\Lambda^{\text{dS}}}{\rho_X}} \sim \frac{\mu_\Lambda^2}{\mu_X^2}, \quad (66)$$

where μ_X depends on the scheme of gravity-mediated SUSY breaking. In the above examples we roughly get the estimate $\theta_K \sim 10^{-(46-48)}$.

VIII. GENERATION PROBLEM

The vacuum states $|\Phi_S\rangle$ and $|\Phi_X\rangle$ are determined by classical values of scalar field in the local minima of its potential. So, the quantization of dynamical fields in vicinity of such vacua are straightforwardly standard. The question is how can we quantize the fields over the true vacuum being the superposition of such two states in accordance with (43)?

First, we can determine the field masses in vacua $|\Phi_S\rangle$ and $|\Phi_X\rangle$, respectively, in ordinary way. Say, let m_S and m_X be the masses of fermion field as given by such the procedure. Hence, the masses corresponds to the cases of exact and broken SUSY.

Second, the superposition of vacuum states is equivalently described by 2D vector or column

$$|\text{vac}\rangle \mapsto \begin{pmatrix} \cos \theta_K \\ \sin \theta_K \end{pmatrix}. \quad (67)$$

Therefore, the mass term of fermion field should be given by 2×2 -matrix of general form

$$M = \begin{pmatrix} m_X & \bar{m} \\ \bar{m} & m_S \end{pmatrix}. \quad (68)$$

It is clear that such the construction is responsible for two generations of the same field.

Thus, the vacuum structure in the form of superposition can be the origin of generations observed in the Standard Model. Then, one should suggest the superposition of three vacuum levels, at least. Probably, one could prefer for the situation with *two flat* vacua and *single AdS* vacuum as it depicted in Fig. 2. Then, the hamiltonian of vacuum contains the mixing of AdS level with *each* flat state $|\Phi_S\rangle_+$ and $|\Phi_S\rangle_-$ at positive and negative values of flat minima, while the eigenstate relevant

to our Universe takes the form of superposition

$$|\text{vac}\rangle_{3G} \approx \frac{1}{\sqrt{2}} \{ |\text{vac}\rangle_+ + |\text{vac}\rangle_- \}, \quad (69)$$

which is represented as 3D vector

$$|\text{vac}\rangle_{3G} \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} 2 \sin \theta_K \\ \cos \theta_K \\ \cos \theta_K \end{pmatrix}, \quad (70)$$

in the basis of states $\{|\Phi_X\rangle, |\Phi_S\rangle_+, |\Phi_S\rangle_-\}$, that could be actual for 3 generations, probably, with some realistic textures of mass matrices of matter fields.

We finalize at this point, since the consideration of spectroscopy is beyond the scope of present paper. The problem is reduced to calculation of non-diagonal “masses” *a la* \bar{m} in (68).

IX. CONCLUSION

In this paper we have described the mechanism for dynamical generation of small cosmological constant due to seesaw mixing of two initial vacuum-states describing the phases of exact and broken supersymmetry. The current value of cosmological constant is consistent with phenomenological estimates of SUSY broken scale in particle physics.

The mechanism works due to fluctuations formed by bubbles of AdS vacuum separated by domain walls from the flat vacuum. We have classified the cases of thin and thick domain walls related with gauge or gravity-mediated SUSY breaking, respectively. The mixing results in the superposition of initial vacua, that could set the origin of three generations of fermions in the Standard Model.

Further studies of such the mechanism have to answer important questions on the spectroscopy of matter and superpartners as well as on a role of mixing angle θ_K and methods of its direct measurement. In addition, one should clarify why we are living in the vacuum we have got. An answer to this question could disfavor the scheme with two flat vacua as presented in Section VIII. Then, an inverse picture with two AdS-vacua and single flat vacuum could be more realistic. This possibility will be investigated elsewhere [21]. Nevertheless, basic features of scale dependence found in the present paper, should remain valid with no changes.

Acknowledgement

The work of V.V.K. is partially supported by the Russian Foundation for Basic Research, grant 07-02-00417.

-
- [1] A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astron. J.* **116**, 1009 (1998) [arXiv:astro-ph/9805201];
B. P. Schmidt *et al.* [Supernova Search Team Collaboration], *Astrophys. J.* **507**, 46 (1998) [arXiv:astro-ph/9805200];
S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517**, 565 (1999) [arXiv:astro-ph/9812133];
J. P. Blakeslee *et al.* [Supernova Search Team Collaboration], *Astrophys. J.* **589**, 693 (2003) [arXiv:astro-ph/0302402];
A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astrophys. J.* **560**, 49 (2001) [arXiv:astro-ph/0104455].
 - [2] A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astrophys. J.* **607**, 665 (2004) [arXiv:astro-ph/0402512].
 - [3] P. Astier *et al.*, arXiv:astro-ph/0510447.
 - [4] D. N. Spergel *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **148**, 175 (2003) [arXiv:astro-ph/0302209];
D. N. Spergel *et al.*, arXiv:astro-ph/0603449.
 - [5] D. J. Eisenstein *et al.*, arXiv:astro-ph/0501171;
S. Cole *et al.* [The 2dFGRS Collaboration], *Mon. Not. Roy. Astron. Soc.* **362**, 505 (2005) [arXiv:astro-ph/0501174].
 - [6] T. Chiba, *Phys. Rev. D* **60**, 083508 (1999) [arXiv:gr-qc/9903094];
N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, *Science* **284**, 1481 (1999) [arXiv:astro-ph/9906463];
P. J. Steinhardt, L. M. Wang and I. Zlatev, *Phys. Rev. D* **59**, 123504 (1999) [arXiv:astro-ph/9812313];
L. M. Wang, R. R. Caldwell, J. P. Ostriker and P. J. Steinhardt, *Astrophys. J.* **530**, 17 (2000) [arXiv:astro-ph/9901388].
 - [7] V. Sahni and A. Starobinsky, *Int. J. Mod. Phys. D* **15**, 2105 (2006) [arXiv:astro-ph/0610026].
 - [8] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
 - [9] I. Y. Kobzarev, L. B. Okun and M. B. Voloshin, *Sov. J. Nucl. Phys.* **20**, 644 (1975) [*Yad. Fiz.* **20**, 1229 (1974)];
S. R. Coleman, *Phys. Rev. D* **15**, 2929 (1977) [Erratum-ibid. **D 16**, 1248 (1977)];
C. G. Callan and S. R. Coleman, *Phys. Rev. D* **16**, 1762 (1977).
 - [10] S. R. Coleman and F. De Luccia, *Phys. Rev. D* **21**, 3305 (1980).
 - [11] S. Weinberg, *Phys. Rev. Lett.* **48**, 1776 (1982).
 - [12] H. Fritzsch, *Phys. Lett. B* **70**, 436 (1977);
H. Harari, H. Haut and J. Weyers, *Phys. Lett. B* **78**, 459 (1978);
H. Fritzsch, *Nucl. Phys. B* **155**, 189 (1979);
Y. Koide, *Phys. Rev. D* **28**, 252 (1983);
P. Kaus and S. Meshkov, *Mod. Phys. Lett. A* **3**, 1251 (1988) [Erratum-ibid. **A 4**, 603 (1989)];
Y. Koide, *Phys. Rev. D* **39**, 1391 (1989);

- H. Fritzsch and J. Plankl, Phys. Lett. B **237**, 451 (1990).
- [13] E. Witten, Nucl. Phys. B **202**, 253 (1982).
 - [14] S. Weinberg, “The quantum theory of fields”, Volume III “Supersymmetry”, Cambridge University Press, 2000.
 - [15] W. Nahm, Nucl. Phys. B **135**, 149 (1978).
 - [16] A. A. Andrianov, F. Cannata, P. Giacconi, A. Y. Kamenshchik and R. Soldati, Phys. Lett. B **651**, 306 (2007) [arXiv:0704.1436 [gr-qc]].
 - [17] T. Banks, arXiv:hep-th/0211160.
 - [18] M. McGuigan, arXiv:hep-th/0602112; hep-th/0604108.
 - [19] K. Enqvist, S. Hannestad and M. S. Sloth, Phys. Rev. Lett. **99**, 031301 (2007) [arXiv:hep-ph/0702236].
 - [20] T. Banks, arXiv:hep-th/0007146.
 - [21] V. V. Kiselev and S. A. Timofeev, in preparation.